Indian Statistical Institute, Bangalore
B. Math. First Year, First Semester
Probability Theory: Final Examination

Date : 10-11-2014 Time: 3 hours
Maximum score: 100

1. Suppose that 20 percent of smokers get lung cancer and 1 percent of non-smokers get lung cancer. In a population there are equal number of males and females, however 30 percent of males are smokers and 20 percent of females are smokers. A person was chosen at random from the population. What is the probability that he/she has lung cancer? If the chosen person has lung cancer, what is the conditional probability that it is a non-smoking male? [20]

2. Let k be a natural number. Consider a coin, where the chance of 'Head' is p with $0 < p \leq 1$. Let G be the number of independent tosses of the coin to obtain k many 'Heads' in succession (i.e., consecutively) for the first time. Show that $P(G = k+1) = P(G = k+2) = \cdots = P(G = 2k)$. [20]

3. Suppose X, Y are independent random variables. Assume that X has Binomial distribution with parameters (m, p) and Y has Binomial distribution with parameters (n, p), where m, n are natural numbers and $0 \leq p \leq 1$. Show that $Z = X + Y$ has Binomial distribution with parameters $(m + n, p)$. Show that $X - Y$ does not have Binomial distribution, even if $m \geq n$. [20]

4. Suppose U is a random variable having uniform distribution in the interval $[-2, 2]$. Compute probability distribution function and densities of $R = \frac{U}{2} + 1$ and $S = U^2 + 1$. [20]

5. Suppose X has Binomial distribution with parameters (n, p), where n is a natural number and $0 < p < 1$. Suppose Y is a random variable defined as a function of X by

$$Y = \begin{cases} 0 & \text{if } X = 0 \\ 1 & \text{otherwise} \end{cases}$$

Compute the conditional distribution of X given $Y = 1$ and the conditional expectation of X given $Y = 1$. [20]

6. Suppose $\lambda > 0$ and E is an exponential random variable with parameter λ. Compute a probability density function for $K = 1 + \sqrt{E}$. [10]